

Practice Test 2

AP® Calculus AB Exam

SECTION I: Multiple-Choice Questions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour and 45 minutes Number of Questions 45 Percent of Total Grade 50% Writing Instrument Pencil required

Instructions

Section I of this examination contains 45 multiple-choice questions. Fill in only the ovals for numbers 1 through 45 on your answer sheet.

CALCULATORS MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question

Sample Answer

 $(A) \odot (D)$

Chicago is a

- (A) state
- (B) city
- (C) country
- (D) continent

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all the multiple-choice questions.

About Guessing

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. Multiple-choice scores are based on the number of questions answered correctly. Points are not deducted for incorrect answers, and no points are awarded for unanswered questions. Because points are not deducted for incorrect answers, you are encouraged to answer all multiple-choice questions. On any questions you do not know the answer to, you should eliminate as many choices as you can, and then select the best answer among the remaining choices.

CALCULUS AB

SECTION I, Part A

Time—60 Minutes

Number of questions-30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

<u>Directions</u>: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

1. If
$$g(x) = \frac{1}{32}x^4 - 5x^2$$
, find $g'(4)$.
(A) -72
(B) -32
(C) 24
(D) 32

2. $\lim_{x \to 0} \frac{8x^2}{\cos x - 1} =$ (A) -16 (B) -1 (C) 8 (D) 6

3.
$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$$
 is

(A) 0

- (B) 5
- (C) 10(D) The limit does not exist.

4. If
$$f(x) = \frac{x^5 - x + 2}{x^3 + 7}$$
, find $f'(x)$.

(A)
$$\frac{(5x^4-1)}{(3x^2)}$$

(B)
$$\frac{(x^3+7)(5x^4-1)-(x^5-x+2)(3x^2)}{(x^3+7)}$$

(C)
$$\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$$

(D)
$$\frac{(x^3+7)(5x^4-1)-(x^5-x+2)(3x^2)}{(x^3+7)^2}$$

5. Evaluate
$$\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}.$$
(A) 0
(B) 1

(B) 1(C) 2

(D) The limit does not exist.

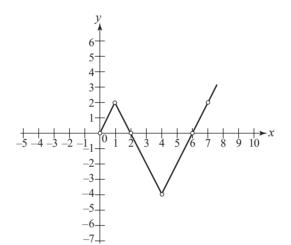
6. $\int x\sqrt{3x} \, dx =$

(A)
$$\frac{2\sqrt{3}}{5}x^{\frac{5}{2}} + C$$

(B) $\frac{5\sqrt{3}}{2}x^{\frac{5}{2}} + C$
(C) $\frac{\sqrt{3}}{2}x^{\frac{1}{2}} + C$
(D) $\frac{5\sqrt{3}}{2}x^{\frac{3}{2}} + C$

- 7. For what value of k is f continuous at x = 1 if $f(x) = \begin{cases} x^2 3kx + 2; x \le 1\\ 5x kx^2; x > 1 \end{cases}$.
 - (A) –1

 - (A) $-\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2
 - (D) 8
- 8. The graph of f(x) is given below. Evaluate $\int_0^7 f(x) dx$.



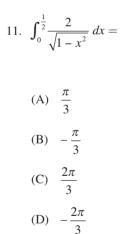
- (A) –11 (B) -4
- (C) 5 (D) 11

9. Find
$$\frac{dy}{dx}$$
 if $y = \sec(\pi x^2)$.

- (A) $\tan(\pi x^2)$
- (B) $(2\pi x)\tan(\pi x^2)$
- (C) $\sec(\pi x^2)\tan(\pi x^2)$
- (D) $\sec(\pi x^2)\tan(\pi x^2)(2\pi x)$

10. Given the curve $y = 5 - (x - 2)^{\frac{2}{3}}$, find $\frac{dy}{dx}$ at x = 2.

- (A) $-\frac{2}{3}$ (B) $-\frac{2}{3\sqrt[3]{2}}$ (C) 5
- (D) The limit does not exist.



12. Let *f* be the function $f(x) = \begin{cases} x^2 - 3bx + 2; x \le 2\\ 2bx^2 - 8; x > 2 \end{cases}$. For what value of *b* is *f* continuous at x = 2? (A) 0 (B) $\frac{1}{4}$ (C) 1 (D) 4

- 13. Let *f* be the function defined by $f(x) = xe^{-x}$. What is the absolute maximum value of *f*?
 - (A) 0
 - (B) <u>1</u>
 - е
 - (C) 1
 - (D) *e*

14. Find
$$\frac{dy}{dx}$$
 at (1, 2) for $y^3 = xy - 2x^2 + 8$.
(A) $-\frac{11}{2}$
(B) $-\frac{2}{11}$
(C) $\frac{2}{11}$
(D) $\frac{11}{2}$

15.
$$\lim_{x \to 0} \frac{x \cdot 2^x}{2^x - 1} =$$
(A) ln 2
(B) 1
(C) 2

(D) $\frac{1}{\ln 2}$

$$16. \quad \int x \sec^2(1+x^2) dx =$$

- (A) $\frac{1}{2}\tan(1+x^2) + C$
- (B) $2\tan(1+x^2) + C$
- (C) $\frac{x}{2}\tan(1+x^2) + C$
- (D) $2x\tan(1+x^2) + C$

- 17. Find the equation of the tangent line to $9x^2 + 16y^2 = 52$ through (2, -1).
 - (A) -9x + 8y 26 = 0
 - (B) 9x 8y 26 = 0
 - (C) 9x 8y 106 = 0
 - (D) 8x + 9y 17 = 0

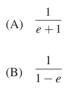
- 18. A particle moves along the *x*-axis. The velocity of the particle at time *t* is given by $v(t) = 12t 3t^2$. What is the total distance traveled by the particle from time t = 0 to t = 6?
 - (A) 0
 - (B) 32
 - (C) 64
 - (D) 96

19. If $f(x) = 3^{\pi x}$, then f'(x) =

(A)
$$\frac{3^{\pi x}}{\ln 3}$$

(B) $\frac{3^{\pi x}}{\pi}$
(C) $\pi (3^{\pi x-1})$

- (D) $\pi \ln 3(3^{\pi x})$
- 20. The average value of $f(x) = \frac{1}{x}$ from x = 1 to x = e is



(C) *e* – 1

(D)
$$\frac{1}{e-1}$$

21. If $y = (x^4 + \sin x)^6$, then $\frac{dy}{dx} =$ (A) $6(x^4 + \sin x)^6(4x^3 - \cos x)$ (B) $6(x^4 + \sin x)^5(4x^3 + \cos x)$ $(\mathbf{C}) \quad 6\left(4x^3 + \cos x\right)^5$

(D)
$$6(4x^3 - \cos x)^5$$

22. Find the slope of the normal line to $y = x + \cos xy$ at (0, 1).

- (A) -1
- (B) 1 (C) 0
- (D) Undefined

23.
$$\int \frac{\csc^2(\sqrt{x})}{\sqrt{x}} dx =$$
(A) $2\cot\sqrt{x} + C$
(B) $-2\cot\sqrt{x} + C$
(C) $\frac{\csc^2\sqrt{x}}{3\sqrt{x}} + C$

(D)
$$\frac{\csc^2 \sqrt{x}}{6\sqrt{x}} + C$$

24.
$$\lim_{x \leftarrow 0} \frac{\tan^3(2x)}{x^3} =$$

(B) 2 (C) 8

$$(\mathbf{C})$$

(D) The limit does not exist.

- 25. A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the *x*-axis, and the *y*-axis is revolved about the *x*-axis. Its volume is found by evaluating which of the following integrals?
 - (A) $\pi \int_0^1 (1 + \sin^4 x) \, dx$
 - (B) $\pi \int_0^1 (1 + \sin^2 x)^2 dx$

(C)
$$\pi \int_{0}^{\frac{\pi}{2}} (1 + \sin^4 x) dx$$

(D) $\pi \int_{0}^{\frac{\pi}{2}} (1 + \sin^2 x)^2 dx$

26. If
$$y = \left(\frac{x^3 - 2}{2x^5 - 1}\right)^4$$
, find $\frac{dy}{dx}$ at $x = 1$.
(A) -52
(B) -28
(C) 13
(D) 52

27.
$$\int x\sqrt{5-x} \, dx =$$
(A) $-\frac{10}{3}(5-x)^{\frac{3}{2}}$
(B) $\frac{10}{3}\sqrt{\frac{5x^2}{2} - \frac{x^3}{3}} + C$
(C) $10(5-x)^{\frac{1}{2}} + \frac{2}{3}(5-x)^{\frac{3}{2}} + C$
(D) $-\frac{10}{3}(5-x)^{\frac{3}{2}} + \frac{2}{5}(5-x)^{\frac{5}{2}} + C$

28. Given the differential equation $\frac{dy}{dt} = -2y$, where y(0) = 100, find y(2).

(A) –200

(B) –4

(C)
$$\frac{100}{e^{16}}$$

(D)
$$\frac{100}{e^4}$$

29. Given $y = x^4 - 6x^3 - 24x^2 + 10$, for which of the following values of *x* does the graph of *y* have a point of inflection?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

30. $\int_{0}^{1} \tan x \, dx =$

- (A) 0
- (B) ln(cos(1))
- (C) $\ln(\sec(1))$
- (D) $\ln(\sec(1)) 1$

END OF PART A, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

<u>Directions</u>: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- 1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- 2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

31. The graph of $y = \frac{1}{2} + \cos x$ has a zero on the interval [0, π]. What is the slope of the tangent line to the graph at that point?

(A)
$$\frac{\sqrt{3}}{2}$$

(B) $-\frac{\sqrt{3}}{2}$
(C) $-\frac{\sqrt{2}}{2}$
(D) $\frac{\sqrt{2}}{2}$

32.
$$\frac{d}{dx} \int_{0}^{x^{2}} \sin^{2} t \, dt =$$

(A) $x^{2} \sin^{2}(x^{2})$
(B) $2x \sin^{2}(x^{2})$
(C) $\sin^{2}(x^{2})$
(D) $x^{2} \cos^{2}(x^{2})$

33. Given $y = x^{\cos 4x}$, find $\frac{dy}{dx}$.

(A)
$$\frac{dy}{dx} = x^{\cos 4x} \left[-\left(\frac{1}{x}\right) (4\sin 4x) \right]$$

(B)
$$\frac{dy}{dx} = x^{\cos 4x} \left[(\cos 4x) \left(\frac{1}{x}\right) - \ln x (4\sin 4x) \right]$$

(C)
$$\frac{dy}{dx} = (\cos 4x)x^{(\cos 4x)-1}$$

(D)
$$\frac{dy}{dx} = \left[(\cos 4x) x^{(\cos 4x) - 1} \right] (-4\sin 4x)$$

- 34. If *f* is defined by $f(x) = x + e^{-x^2}$ on the interval [0, 10], then *f* has a point of inflection at which of the following values of *x* ?
 - (A) 0.379
 - (B) 0.5
 - (C) 0.707
 - (D) 0.947

35. Estimate $\int_0^2 3e^x + 1 \, dx$ using a Riemann sum with n = 4 right-hand rectangles.

- (A) 26.357
- (B) 33.546
- (C) 52.713
- (D) 56.713

- 36. The volume generated by revolving about the *x*-axis the region above the curve $y = x^3$, below the line y = 1, and between x = 0 and x = 1 is
 - (A) $\frac{\pi}{42}$
 - (B) 0.143*π*
 - (C) 0.643π
 - (D) $\frac{6\pi}{7}$

- 37. A sphere is increasing in volume at the rate of $20 \frac{\text{in.}^3}{\text{s}}$. At what rate is the radius of the sphere increasing when the radius is 4 in. ?
 - (A) $0.025 \frac{\text{in.}}{\text{s}}$ (B) $0.0995 \frac{\text{in.}}{\text{s}}$ (C) $0.424 \frac{\text{in.}}{\text{s}}$ (D) $0.982 \frac{\text{in.}}{\text{s}}$

- 38. The function *f* is given by $f(x) = 2x^3 5$ on the interval [1, 5]. Which of the following is guaranteed by the Mean Value Theorem for some value *c* on the interval (1, 5) ?
 - (A) 3.136
 - (B) 3.215
 - (C) 3.225
 - (D) 4.160

- 39. Find two non-negative numbers x and y whose sum is 100 and for which x^2y is a maximum.
 - (A) x = 50 and y = 50
 - (B) x = 33.333 and y = 66.667
 - (C) x = 100 and y = 0
 - (D) x = 66.667 and y = 33.333

- 40. An object is moving along a line with its velocity given by $v(t) = t^2 \sin t$, for time $t \ge 0$. If the object's position at time t = 0 is 4, what is its position at time t = 2?
 - (A) 0.469
 - (B) 2.469
 - (C) 4.469
 - (D) 6.469

41. $\int \sin^4(\pi x) \cos(\pi x) dx =$

(A)
$$\frac{\sin^5(\pi x)}{5\pi} + C$$

(B)
$$\frac{\sin^5(\pi x)}{2\pi} + C$$

(C)
$$-\frac{\cos^5(\pi x)}{5\pi} + C$$

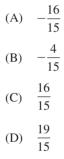
(D)
$$-\frac{\cos^2(\pi x)}{2\pi} + C$$

42. A balloon is inflating at the rate $\frac{dV}{dt} = 300 - t \ln t \frac{\ln^3}{s}$, where *t* is the number of seconds that the balloon has been inflating. If the initial volume of the balloon is 100 in³, what is the volume of the balloon after it has been inflating for

8 seconds, to the nearest 10 in.³?

- (A) 150 in.³
- (B) 280 in.³
- (C) 320 in.³
- (D) 2450 in.³

43. If $x^2 + 3x^2y + y^3 = 13$, find $\frac{dy}{dx}$ at (2, 1).



- 44. Find the equation of the line tangent to $y = x \tan x$ at x = 1.
 - (A) y = 4.983x + 3.426
 - (B) y = 4.983x 3.426
 - (C) y = 4.983x + 6.540
 - (D) y = 4.983x 6.540

45. If f(x) is continuous and differentiable and $f(x) = \begin{cases} ax^4 + 5x; x \le 2\\ bx^2 - 3x; x > 2 \end{cases}$, then $b = bx^2 - 3x$.

- (A) 0
- (B) 2

(C) 6

(D) There is no value of b.

STOP

END OF PART B, SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY. DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

SECTION II GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2 dx$ may not be written as fnInt (X², X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

SECTION II, PART A Time—30 minutes Number of problems—2

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B \cos\left(\frac{\pi H}{12}\right)$, where *T* is the temperature in

degrees Fahrenheit and *H* is the number of hours from midnight ($0 \le H < 24$).

- (a) The initial temperature at midnight was $-15^{\circ} F$ and at noon of New Year's Day was $5^{\circ} F$. Find A and B.
- (b) Find the average temperature for the first 10 hours.
- (c) Use the Trapezoid Rule with 4 equal subdivisions to estimate $\int_{-\infty}^{\infty} T(H) dH$.
- (d) Find an expression for the rate that the temperature is changing with respect to H.
- 2. Sea grass grows on a lake. The rate of growth of the grass is $\frac{dG}{dt} = kG$, where k is a constant.
 - (a) Find an expression for *G*, the amount of grass in the lake (in tons), in terms of *t*, the number of years, if the amount of grass is 100 tons initially and 120 tons after one year.
 - (b) In how many years will the amount of grass available be 300 tons?
 - (c) If fish are now introduced into the lake and consume a consistent 80 tons/year of sea grass, how long will it take for the lake to be completely free of sea grass?

SECTION II, PART B Time—1 hour Number of problems—4

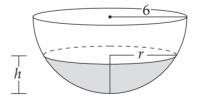
No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

3. The functions f and g are twice-differentiable and have the following table of values:

x	f(x)	f'(x)	g(x)	g'(x)
1	3	4	-2	-4
2	2	3	4	-2
3	5	2	-1	3
4	-1	-6	-8	0

- (a) Let h(x) = f(g(x)). Find the equation of the tangent line to *h* at x = 2.
- (b) Let j(x) = f(x)g(x). Find j'(3).
- (c) Evaluate $\int_{1}^{4} 3f''(x) dx$.
- 4. Water is being poured into a hemispherical bowl of radius 6 inches at the rate of 4 in.³/sec.



(a) Given that the volume of the water in the spherical segment shown above is $V = \pi h^2 \left(R - \frac{h}{3} \right)$, where *R* is the radius of

the sphere, find the rate that the water level is rising when the water is 2 inches deep.

- (b) Find an expression for *r*, the radius of the *surface of the spherical segment* of water, in terms of *h*.
- (c) How fast is the circular area of the surface of the spherical segment of water growing (in in.²/sec) when the water is 2 inches deep?

- 5. Let *R* be the region in the first quadrant bounded by $y^2 = x$ and $x^2 = y$.
 - (a) Find the area of region R.
 - (b) Find the volume of the solid generated when R is revolved about the x-axis.
 - (c) The section of a certain solid cut by any plane perpendicular to the *x*-axis is a circle with the endpoints of its diameter lying on the parabolas $y^2 = x$ and $x^2 = y$. Find the volume of the solid.
- 6. For time $t \ge 0$, a particle moves along the *x*-axis. The velocity of the particle at time *t* is given by $v(t) = 1 2\cos\left(\frac{\pi}{3}t\right)$. The particle's position at time t = 0 is x(0) = 8.
 - (a) Is the particle speeding up or slowing down at time $t = \frac{1}{2}$? Justify your answer.
 - (b) When does the particle change direction in the interval $0 \le t \le 2$? Justify your answer.
 - (c) What is the particle's position at time t = 2?
 - (d) What is the total distance traveled from time t = 0 to time t = 2?

STOP

END OF EXAM

Princeton **Review**[®] Completely darken bubbles with a No. 2 pencil. If you make a mistake, be sure to erase mark completely. Erase all stray marks. 5. YOUR NAME 1. First 4 letters of last name MI _____ DATE: ____ / / SIGNATURE: BBBB \bigcirc \mathbb{D} \mathbb{D} \mathbb{D} \mathbb{D} City State Zip Code EEE ĐĐ ĐĐ PHONE NO.: \bigcirc IMPORTANT: Please fill in these boxes exactly as shown on the back cover of your test book. HHHH 2. TEST FORM 3 TFST CODE \bigcirc 4 REGISTRATION NUMBER \bigcirc

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